TABLE IV

SUMMARY OF EVEN-MODE AND ODD-MODE IMPEDANCE VALUES FOR THE FILTERS OF FIG. 8(a)–(c) DESIGNED BY USE OF TABLE I AND REALIZED IN THE FORM IN FIG. 1(a)

	Fig. 8(a) (5% Bandwidth)	Fig. 8(b) (30% Bandwidth)	Fig. 8(c) (2 to 1 Bandwidth)
$(Z_{oe})_{01}=(Z_{oe})_{67}$	1.251	1.540	1.716
$(Z_{oe})_{12}=(Z_{oe})_{56}$	0.996	1.023	1.142
$(Z_{oe})_{23}=(Z_{oe})_{45}$	0.981	0.937	0.954
$(Z_{oe})_{34}$	0.980	0.927	0.933
$(Z_{oo})_{01}=(Z_{oo})_{67}$	0.749	0.460	0.284
$(Z_{oo})_{12}=(Z_{oo})_{56}$	0.881	0.491	0.208
$(Z_{oo})_{23}=(Z_{oo})_{45}$	0.895	0.536	0 250
$(Z_{oo})_{34}$	0.896	0.542	0.255

All values normalized so that $Z_o=1$.

TABLE V

ELEMENT VALUES FOR THE FILTER OF FIG. 9 REALIZED AS SHOWN IN FIG. 1(d)

Filter designed using Table II from a 0.10-db ripple, $n=8$, Tchebycheff prototype using $\omega_1/\omega_0=0.650$.

$Y_1=Y_8=1.042$	$Y_3=Y_6=2.049$
$Y_{12}=Y_{78}=1.288$	$Y_{34}=Y_{56}=1.292$
$Y_2=Y_7=2.050$	$Y_4=Y_5=2.087$
$Y_{23}=Y_{67}=1.364$	$Y_{45}=1.277$

All values normalized so $Y_0=1$.

TABLE VI

ELEMENT VALUES FOR THE FILTER OF FIG. 10 REALIZED AS SHOWN IN FIG. 2

Filter designed from a 0.10-db ripple, $n=8$, Tchebycheff prototype using $\omega_1/\omega_0=0.850$ and $\omega_\infty/\omega_0=0.500$. This, then, calls for $a=1$ so that $Y_k'=Y_k''$ throughout.

$Y_1'=Y_8'=1.806$	$Y_3'=Y_6'=3.584$
$Y_{12}=Y_{78}=1.288$	$Y_{34}=Y_{56}=1.292$
$Y_2'=Y_7'=3.585$	$Y_4'=Y_5'=3.614$
$Y_{23}=Y_{67}=1.364$	$Y_{45}=1.277$

All values normalized so that $Y_0=1$.

TABLE VII

ELEMENT VALUES FOR THE FILTER OF FIG. 11 REALIZED AS SHOWN IN FIG. 3

Filter designed using Table III from a 0.10-db ripple, $n=8$, Tchebycheff prototype using $\omega/\omega_0=0.650$ and $\omega_\infty/\omega_0=0.500$.

$Z_1=Z_8=0.606$	$Y_3=Y_6=1.235$
$Z_1'=Z_8'=0.606$	$Y_{34}=Y_{56}=0.779$
$Y_2=Y_7=1.779$	$Y_4=Y_5=1.258$
$Y_{23}=Y_{67}=0.823$	$Y_{45}=0.770$

Radio-Frequency System of the Cambridge Electron Accelerator*

KENNETH W. ROBINSON†, MEMBER, IRE

Summary—**The requirements for the RF system of the Cambridge Electron Accelerator are investigated and the choice of the major parameters of the system is discussed. The strongly coupled waveguide cavity system is analyzed and the performance of the system with various types of imperfections is calculated.**

INTRODUCTION

THE Cambridge Electron Accelerator is a project to design and construct a 6-billion-volt electron synchrotron at Harvard University to be used for high energy physics research.[1]

The Cambridge accelerator has two important differences from most synchrotrons which require the design of the RF system to be very different from that of other circular accelerators. The electrons radiate electromagnetic energy due to the curvature of their orbits in the magnetic guide field. At 6 bev, the radiation loss is 4.5 mev per turn. This radiation loss occurs as discrete quanta which are typically 15-kv X rays at 6 bev. The emission of these discrete quanta produces synchronous oscillations of the individual electrons in energy and phase position about the equilibrium values, and requires the RF voltage to be substantially larger than the radiation loss, in order to prevent the particles being lost from the phase stable region.[2] The other characteristic is the initial injection of electrons at an energy of 20 mev or higher which eliminates the need to modulate the frequency, and makes it possible to use high Q cavities for the system.

The problem of the RF system is, then, to design a system capable of developing the large accelerating voltage required to make up the peak radiation loss of 4.5 mev per turn and contain the quantum induced phase oscillations. During the acceleration cycle the radiation

* Received by the PGMTT, April 14, 1960; revised manuscript received July 13, 1960.

† Cambridge Electron Accelerator, Harvard University, Cambridge, Mass.

[1] *Proc. Internatl. Conf. on High-Energy Accelerators and Instrumentation*, CERN, Geneva, Swiz., pp. 335–338; 1959.

[2] M. Sands, "Synchroton oscillations induced by radiation fluctuations," *Phys. Rev.*, vol. 97, pp. 470–473; 1955.

loss is proportional to the fourth power of the electron energy, so the RF power required during the early portion of the acceleration cycle will be much less than the peak RF power. The ratio of average to peak RF power required is estimated to be about 15 per cent.

The magnet structure of the Cambridge accelerator has 48 magnets with 48 equal-length straight sections. The number of RF cavities is chosen to be 16, as the largest number of symetrically located cavities which can be put into the 48 straight sections, due to other requirements for use of the straight sections.

The choice of the frequency of the RF system is determined by several factors. The general range of the frequency is chosen so as to approximately minimize the RF power required. In a cavity of fixed total length, the number of cavity sections may be increased proportionally to the frequency, and the total shunt impedance will then increase porportionally to the square root of the frequency. However, as the frequency is increased and the wavelength is decreased, the allowable longitudinal displacement in a phase oscillation is reduced, and a higher RF accelerating voltage is required to prevent the electrons from being lost due to the quantum induced synchronous oscillations. Quantitative evaluation of these two factors gives an optimum frequency of about 600 mc. Other considerations, such as the aperture required in the cavity for the electrons, and the desire to synchronize the RF system with the linear accelerator used for injection, lead to a choice of 475 mc as the operating frequency.

The calculated losses of electrons due to the quantum induced synchronous oscillations are shown in Fig. 1. With a peak RF voltage of 6 mev per turn, less than 10 per cent of the electrons will be lost before reaching 6 bev. If the peak RF voltage is only 5 mev per turn, most of the electrons will be lost before reaching 6 bev.

The design of the RF cavity is shown in Fig. 2. The cavity consists of two half-wave sections partially loaded with re-entrant drift tubes to reduce the transit time factor of the electrons traversing the cavity. Each cavity has an inductive aperture with a ceramic window for coupling to waveguide. There is also an inductive slot in the center wall for coupling the two half-sections of a single cavity. The shunt impedance of a single cavity is about 7.5 megohms and the Q is about 25,000. The peak power dissipated in the cavity is about 15 kw and the average power is about 2.2 kw.

In order to supply the RF power to the separate cavities, it is proposed to couple the 16 cavities together by means of waveguide so as to form a closed ring. The dimensions of the waveguide are 9 inches$\times$18 inches, and the length of waveguide between adjacent cavities is about 50 feet.

The waveguide-cavity system will be a standing wave system with many separate modes. By operating at the frequency of the proper mode, the fields in the individual cavities will have the correct phase relationship to accelerate particles.

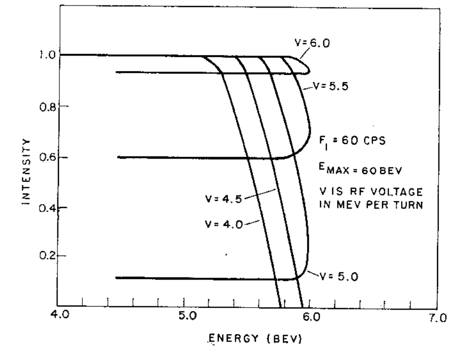

Fig. 1—Loss of particles due to quantum induced synchronous oscillations.

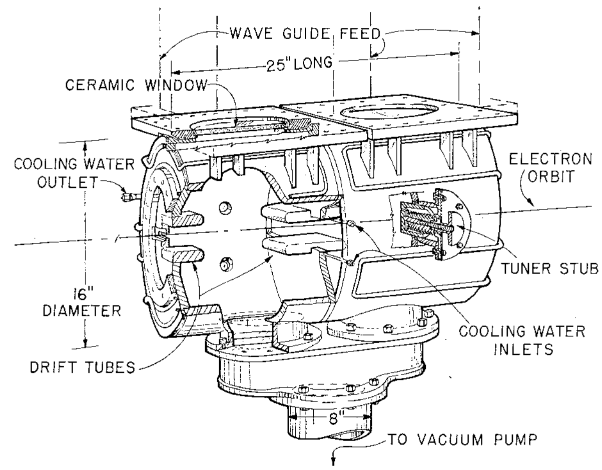

Fig. 2—RF cavity.

The alternative to the strongly coupled system would be to supply the RF power individually to each cavity by means of a transmission line which is matched by the cavity. Since the Q of the cavity is about 25,000, it would be necessary to maintain the tuning of each individual cavity correct to an accuracy of about one part in 100,000. This accuracy would probably require an automatic system on each cavity to maintain the tuning by monitoring the reflected power. The automatic tuning system would be complicated by the effect of the circulating electron beam in the synchrotron, which would cause the reflected power to vary during the acceleration cycle. With the strongly coupled RF system, the tolerance on the accuracy of tuning of each cavity becomes much larger and it is not necessary to control the tuning of each cavity independently; but only to control the tuning of the system as a whole, which may be easily done by controlling the frequency.

Analysis of Strongly Coupled System

In order to analyze the performance of the strongly coupled system, the system is represented by a closed loop of transmission line, coupled to equally spaced resonant cavities as shown in Fig. 3. We assume that the coupling between the two sections of a single cavity is sufficiently large so that in the frequency region of

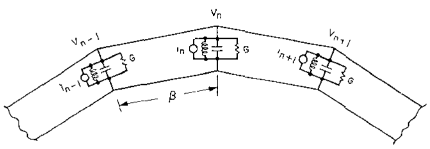

Fig. 3—Schematic representation of strongly coupled RF system.

interest the cavity may be represented by a simple resonant circuit. The length of the transmission line is defined from the principal planes of the cavity. G is the normalized conductance of the cavity. $\gamma=\alpha+i\beta$ is the complex phase angle of the length of transmission line between cavities.

In order to analyze the operation of the strongly coupled system, it is convenient to choose a number of normal spatial modes for the system equal to the number of resonant cavities. The modes are defined so that

$$v_{p,n+1} = v_{p,n}e^{i\theta_p}$$

$$\theta_p = \frac{2\pi p}{N}$$

or

$$i_{p,n+1} = i_{p,n}e^{i\theta_p}.$$

$v_{p,n}$, $i_{p,n}$ is the voltage or current generator at the nth cavity due to mode p. $N=16$ is the total number of cavities or modes. Only one mode will be effective in accelerating electrons. The voltages in the other modes result in increased power dissipation, as compared with an ideal system.

A characteristic admittance may be defined for each mode for the ideal system.

$$Y_p = \frac{i_{p,n}}{v_{p,n}}\,.$$

From the definition of the normal modes and the representation of the strongly coupled system, the characteristic admittance of the modes is calculated to be

$$Y_p = G + 2iGQ\frac{\Delta\omega}{\omega_0} + 2\coth\gamma - (e^{-i\theta_p} + e^{i\theta_p})\operatorname{csch}\gamma.$$

For good performance of the system, it is desirable that the admittance of all the undesired modes be large compared with that of the proper mode, so that imperfections in the system will not cause large voltages and power dissipation in the undesired modes. For this reason, the proper mode is chosen to be a nondegenerate mode with $\cos\theta_p=\pm 1$. For other modes there would be a degenerate mode with $\theta_{p2}=-\theta_{p1}$, with equal admittance. We take the fundamental mode to be $\theta_p=0$ in this analysis. By using the relation $\alpha\ll 1$, the admittance of the fundamental mode becomes

$$Y_0 = G + \frac{2\alpha}{1+\cos\beta} + i2GQ\frac{\Delta\omega}{\omega_0} + i\frac{2\sin\beta}{1+\cos\beta}\,.$$

In order to make the admittance of the undesired modes large, $\beta\ll 1$. Then the admittance of the fundamental mode when the system is tuned to resonance becomes

$$Y_0 = G_0 = G + \alpha.$$

The ratio α/G then represents the fractional increase in power required for the perfect strongly coupled system.

The admittance of the undesired modes with the assumptions $\alpha\ll 1$, $\beta\ll 1$, becomes

$$Y_p = Y_0 + \frac{4\sin^2\frac{\theta_p}{2}}{\alpha^2+\sin^2\beta}(\alpha - i\sin\beta).$$

In order to determine the effect of imperfections on the strongly coupled system we analyze the effect of detuning a single cavity. If the changes in cavity voltages produced by the imperfection are small compared with the fundamental mode voltage, the result of the imperfection may be calculated as a first-order effect. The detuning of a single cavity may be considered equivalent to adding a current generator of $I=iV_0\cdot 2GQ(\Delta\omega/\omega_0)$ at that cavity. This current generator at one cavity may be expressed as a combination of the normal modes of current generators applied to all cavities.

The fundamental mode component of this current is $i(V_0/N)\cdot 2GQ(\Delta\omega/\omega_0)$. The effect of the detuning on the admittance of the fundamental mode can be eliminated by changing the resonant frequency of all the cavities by an amount $-\Delta\omega/N$, or by changing the frequency by an amount $\Delta\omega/N$.

The change in tuning of a single cavity also produces current generators in every undesired mode with a magnitude equal to $(V_0/N)\cdot 2GQ(\Delta\omega/\omega_0)$. The magnitude of the voltage produced in an undesired mode is then

$$V_p = 2\frac{V_0}{N}\frac{GQ}{Y_p}\frac{\Delta\omega}{\omega_0}\,.$$

The power dissipated in an undesired mode is

$$P_p = V_0^2\left(\frac{1}{N}\,2Q\,\frac{\Delta\omega}{\omega_0}\right)^2\frac{G^2}{Y_p^2}G_p.$$

We make the assumption that $\alpha\ll\beta\ll 1$. Then

$$Y_p \approx i4\frac{\sin^2\frac{\theta_p}{2}}{\sin\beta} + G_0 \approx i4\frac{\sin^2\frac{\theta_p}{2}}{\sin\beta} \qquad \text{for } G\ll 1$$

$$G_p \approx 4\frac{\sin^2\frac{\theta_p}{2}}{\sin^2\beta}\alpha + G_0.$$

The ratio of power dissipated in all the undesired modes, due to detuning of a single cavity, to the power dissipated in an ideal system is given by

$$\frac{\Delta P}{P} = \left(2Q\frac{\Delta\omega}{\omega_0}\right)^2 \frac{G}{16N^2}\left\{4\alpha\sum_{p=1}^{15}\frac{1}{\sin^2\frac{\theta_p}{2}} + G_0\sin^2\beta\sum_{p=1}^{15}\frac{1}{\sin^4\frac{\theta_p}{2}}\right\}.$$

The summation over p from 1 to 15 includes all the modes, except the fundamental mode with $p=0$. The summations are evaluated to be

$$\sum_{p=1}^{15}\frac{1}{\sin^2\frac{\theta_p}{2}} = 84.6 \quad\text{and}\quad \sum_{p=1}^{15}\frac{1}{\sin^4\frac{\theta_p}{2}} = 1499.$$

If all the cavities are detuned in a random manner by a rms amount $\Delta\omega/\omega_0$, the power dissipated in the spurious modes will be increased by a factor of N. The fractional total increase in power dissipated in the strongly coupled system, including the power dissipated in the transmission line in the fundamental mode, is then given by

$$\frac{\Delta P_T}{P} = \frac{\alpha}{G} + \left(2\frac{\Delta\omega}{\omega_0}Q\right)^2 \frac{G}{256}\{345\alpha + 1499G\sin^2\beta\}.$$

The value of α for the waveguide transmission line between adjacent cavities is about 0.004. Then

$$\frac{\Delta P_T}{P} = \frac{0.004}{G} + [0.0054G + 5.86(\sin^2\beta)G^2]\left(2Q\frac{\Delta\omega}{\omega_0}\right)^2.$$

If we assume that we want to allow a rms detuning of the individual cavities by 10 half-bandwidths, and take $\sin\beta=0.1$, then the minimum increased power dissipation occurs for $G=0.06$ and is equal to $\Delta P_T/P=0.12$. The fractional power dissipation in the transmission lines for an ideal system with no cavity detunings is then 0.066.

There will also be a possible mode in which the fields are zero in the cavities and all the energy is stored in the transmission lines. This mode will not be excited by detuning of the individual cavities, but may be excited by imperfections in the transmission lines. The worst possible case would be if an imperfection occurred at a point midway between a voltage maximum and current maximum for the fundamental mode on every transmission line. The admittance of this waveguide mode at this point is given approximately by $Y_w \approx i2\beta$. An imperfection in the form of a susceptance of iS at this point can be considered a current generator of magnitude $(V_0/\sqrt{2})S$, which will produce a voltage in the waveguide mode of $V_0S/2\beta$. The power dissipated due to this mode will be

$$\Delta P_w = V_0^2\frac{S^2}{4\beta^2}\alpha$$

and, when expressed as a fraction of the power dissipated in the fundamental modes, is:

$$\frac{\Delta P_w}{P} = \frac{S^2}{4\beta^2}\frac{\alpha}{G}.$$

If we allow an imperfection which will produce a VSWR of 1.1 for the transmission line ($S=0.1$), and take $\beta=0.1$ as before, then

$$\frac{\Delta P_w}{P} = \frac{1}{4}\frac{\alpha}{G} = 0.016 \qquad \text{for } \alpha = 0.004,\ G = 0.06.$$

The RF power is supplied to the standing-wave ring system from a power source in a central location by means of a transmission line which makes a matched junction with the ring at one point. For optimum operation of the system, the junction should be at a cavity or approximately an integral number of half wavelengths from a cavity; which, in the strongly coupled system, is almost electrically equivalent to having the junction at the cavity. The effect of introducing power into the system at only one point may be analyzed in terms of power dissipation in the undesired modes, in a manner similar to the effect of detuning of cavities. With the junction at the optimum position, the waveguide mode will not be excited. The junction may be taken equivalent to a current generator of magnitude NGV at that point. This is equivalent to detuning a single cavity by an amount $2Q(\Delta\omega/\omega_0)=N$, and will then produce an increased power dissipation equal to that produced by detuning all the cavities by a rms amount $2Q(\Delta\omega/\omega_0)=N^{1/2}$. Since $N=16$, this will then result in an increased power dissipation of 16 per cent of that produced by the random detuning of all the cavities by a rms amount of $2Q(\Delta\omega/\omega_0)=10$, which is about 1 per cent of the power dissipated in the fundamental mode.

The power level in the waveguide transmission line may be calculated from the ratio of power dissipated in the waveguide to the power dissipated in the cavities. This ratio is given by

$$\frac{\Delta P}{P} = \frac{0.004}{G} + 0.0054\left(2Q\frac{\Delta\omega}{\omega_0}\right)^2 \text{ and}$$

$$\frac{\Delta P}{P} \approx 0.10.$$

Since the voltage attenuation in the waveguide is 0.004 nepers, this corresponds to a power flow in one direction of about 12.5 times the power dissipated in one cavity. From the previous values of 15 kw peak and 2.2 kw average for the power dissipated in a single cavity, the corresponding power flow in the waveguide becomes 187 kw peak and 27.5 kw average. Since the fields in the waveguide are more like standing wave fields, the maximum fields in the waveguide will correspond to power levels of twice the above values. These power levels in the waveguide will be somewhat further increased, due to the effect of the circulating electron beam in the synchrotron.